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Vertical Instability of One-Zone Accretion Disks

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1. We study the vertical instability of geometrically thin one-zone accretion discs briefly. The target of our study, nonlinear behavior of the simplified model, will be investigated more precisely in a future work.

2. The cylindrical coodinate (R, ϕ, z) is used to describe the disc. We note the mean density of an annule at a given radius R as $\rho(R, t)$. We use h to express the half-thickness of the disc, and m, a half of the surface-density. P is the representative value of the pressure. The circular motion in the disc is expressed by the Keplerian motion. ω is the angular velocity at the radius R. We choose here the standard α-disc model like as usual authors (see Shakura and Sunyaev 1973, 1976; Pringle 1981).

3. We define here the deviations form the equilibrium values, x, u, and w.

$$h = h_0(1+x), \qquad T = T_0(1+u), \qquad m = m_0(1+w). \tag{1}$$

Radiative opacity is assumed as the form:

$$\kappa = \kappa_0 \left(\frac{\rho}{\rho_0}\right)^n \left(\frac{T}{T_0}\right)^{-s}. \tag{2}$$

We assume here that the kinematic viscosity depends on the temperature, that is

$$\alpha = \alpha_0(1+u)^p. \tag{3}$$

The specific heat is assumed as constant for simplicity.

Gas-pressure dominant disc

First, we consider the gas-pressure dominant annuli ($\beta_0 \approx 1$). The simple harmonic oscillation with frequency ω^2 is shown.

When we ignore the changes of w,

$$\frac{7}{2} + s - p > 0, \tag{4}$$

is the condition of thermal instability. The system is vibrationally unstable when $n > 0$.

$$-1 + \frac{n}{2} + (3 + s - p) > 0. \tag{5}$$

This is a condition necessary to keep the discs dynamically stable in the non-adiabatic case.

If we assume that $\omega^2 \gg 1$ and $n < 0$, the condition ,

$$x - \frac{1}{2}u \approx 0, \tag{6}$$

becomes kept approximately. We can find the stability coditions.

Radiation-pressure dominant disc

Next, we study the radiation pressure dominant annuli. When we ignore the variation of u and w, vertical oscillations of the frequency $\Omega/\sqrt{2}$ is found.

By using the Hurwitz theorem, we can see the following conditions for the stability:

$$\begin{aligned} 2 + s - p &> 0, & (7)\\ \frac{3}{2} - n &> 0, & (8)\\ -4(1-n) + s - p &> 0. & (9) \end{aligned}$$

The first of these conditions is that of the thermal instability. The second condition expresses the vibrational stability. Third one is necessary for the stability against the motion without vertical oscillation. Since the radiation-pressure dominant state is hot and rarefied, the electron scattering is main agent of the opacity. Then we may assume that $s = 0$ and $n = 0$. This does not melt into the stability condition with the condition, $p = 0$. As well known, the hot and rarefied discs are dynamically unstable. The instability have been discussed by Lightman and Eardley (1974).

4. The instability of hot and rarefied discs is difficult to remove, but the increase of the derivatives of physical quantities will be restricted by the nonlinear effect. With the dcrease of u, the gas pressure becomes more important against the radiation pressure. This keeps the discs in a finite thickness. In the other case that u increases higher and higher, the radiation pressure becomes more important. The discs should become to very rarefied state and quit from the optically thick discs assumed in this note. When the radiation escapes smoothly from the matter, the radiation pressure losses its significance. Then the discs come back to the gas-pressure dominant state, and become stable. To show the details of such a limit-cycle-like behavior will be shown by numeirical simulation. The detailed studies of nonlinear motion with realistic parameters will be performed in a future paper.

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